Announcements

- HW6 out due Friday as usual
- Hw7 single question, due Tuesday March 17th max 1 late day

- Prelim 2: Tuesday March 24th
  - The conflicts survey is open, due on Monday, March 16th
  - Topics: stable matching, flows and applications and NP-completeness
  - Info sheet on topics and sample question in canvas

- Mid-term survey on sections (email from me with the link)

closing tonight

NP-complete Problem with numbers

Knapsack n items value v_i weight w_i & limit W

find subset I items $\sum_{i \in I} w_i \le W$, max value $\sum_{i \in I} v_i$

We had a dynamic programming algorithm assuming w_i all integers

$O(nW)$ time

? is this polynomial time ?

polynomial time = time $O(\text{input size}^c)$ for some constant c

input size of knapsack problem

$2n+1$ numbers $\max(\max_i w_i, \max_i v_i, W) = W_{max}$

input $O(n \log_2 W_{max})$ written in binary

comment in $O(\cdot)$ base of log does not matter

Subset Sum and Knapsack

Claim Knapsack NP-hard with big number (not in NP)

Subset sum $w_1, \dots, w_n$ & W

Question is a there a subset with W as the sum

Claim Subset Sum $\leq_p$ Knapsack

indeed take knapsack $v_i = w_i$

is max value $= W$?

Claim Subset Sum NP-complete

1. Subset Sum in NP

given subset $I \subseteq 1, \dots, n$ need to check if $\sum_{i \in I} w_i = W$?

takes $O(n \log w_{max})$

adding $w_i + w_j$

123456
321987
43

$O(\log w)$

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To prove SAT $\leq_P$ SUBSET SUM which of the following should we do

A: take input with subset sum and construct an equivalent SAT formula

SAT can be used to solve subset sum

SAT is powerful

B: take input with SAT problem and construct an equivalent subset sum input

3-SAT $\leq_P$ SUBSET SUM: the idea

(using from book 3-SAT NP-complete)

input 3-SAT $\phi = (x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee x_2 \vee x_4) \wedge \cdots$

decisions x_i T or F

subset sum input $w_i = ????\underbrace{0..\overset{i}{1}0..0}_{\text{last } n \text{ digits}}$ $w_{i+n} = ????\underbrace{0\ \overset{i}{1}\ 0..0}$

$w_i = 10^{i-1} + ?\,10^n$ $w_{i+n} = 10^{i-1} + ?\,10^n$

$W = ??...?\underbrace{1...1}$ $W = ?\,10^n + \sum_{i=0}^{n-1} 10^i$

x_1 ..... $w_1 = ???\,0...01$ $\bar{x}_1$ ~ . $w_{1+n} = ??...?\,000..01$

x_2 $w_2 = ???\,0..010$ $\bar{x}_2$ ~ $w_{2+n} = ?...?\,0..010$

$\vdots$

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Number of ways to get a subset if numbers so far to get the last n digits right

A: none

B: 1

C: n

D: 2^n

E: none of these

last n digits

$w = ??\ldots?\underbrace{1\ldots1}$

$x_1 \quad w_1 = ???0\ldots01 \qquad \bar{x}_1 \quad w_{1+n} = ??\ldots?000\ldots01$

$x_2 \quad w_2 = ???0\ldots010 \qquad \bar{x}_2 \quad w_{2+n} = ?\ldots?0\ldots010$

$\vdots$

example $n=3$

??? 001 ??? 001

??? 010 ??? 010

??? 100 ??? 100

need to take w_i or w_{i+n} all i

idea $x_i = T$ if w_i

$x_i = F$ if w_{i+n}

3-SAT $\leq_P$ SUBSET SUM: the construction

$$w_i = 10^{i-1} + \sum_{\substack{j:\text{ clause } j \\ \text{contains } x_i}} 10^{n+j}$$

$$w_{i+n} = 10^{i-1} + \sum_{\substack{j:\text{ clause } j \\ \text{contains } \bar{x}_i}} 10^{j+n}$$

e.g. $w_1 = 10^0 + 10^{n+1} + \ldots$

first clause containing x_1 is clause 1

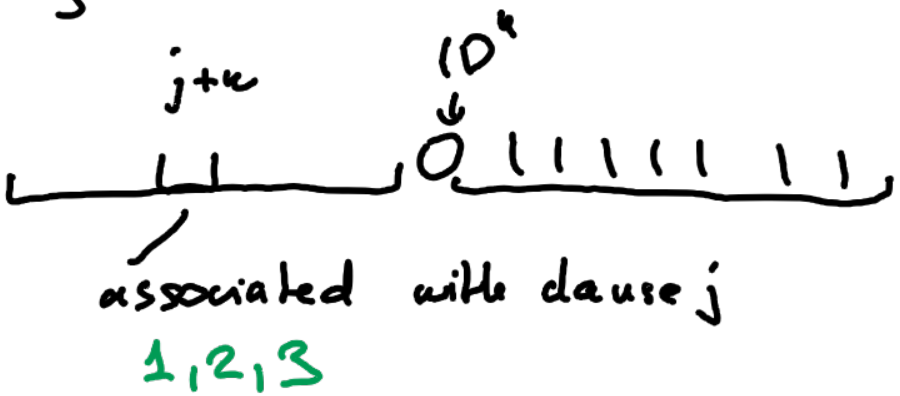
$w_{1+n} = 10^0 + 10^{n+2}$

first clause containing $\bar{x}_1$ is clause 2

Suppose ϕ satisfiable & we pick w_i or w_{i+n} depending on satisfying assignment

What is the sum?

position $n+j$ has 1 in
w_i if x_i in clause
w_{i+n} if $\bar{x}_i$ in clause



add extra numbers:
2 copies each of $\underline{10^{n+j}}$

2 for each clause

$$W = \underbrace{33\ldots3}_{m}\,0\,\underbrace{1111\ldots1}_{n}$$

$$W = \sum_{i=1}^{n} 10^{i-1} + 3\sum_{j=1}^{m} 10^{n+j}$$

input w number $2n+2m$

Claim: this input for subset sum is equivalent to formula being satisfiable

(4) if ϕ satisfiable $\Longrightarrow$ subset sum answer is yes

x_i = T or F satisfying assignment $\longrightarrow$ pick w_i if $x_i = T$
w_{n+i} if $\overline{x}_i = T$

poll question sum last last digits right $? \ldots ?\,0\,\underbrace{1111}_{n \text{ digits}}$

add copies of 10^{n+j} to make that digit 3 = # copies = 3 - # true variables in clause j

Proving construction correct

5 if ϕ not satisfiable $\Longrightarrow$ no way to make sum $= W$

equivalent : if sum can be W $\Longrightarrow$ ϕ satisfiable

$W \quad \underbrace{3 \quad \quad 3}_{\text{need to satisfy each clause}_j} \; 0 \; 1 \cdots 1$

imply valid : w_i or w_{i+n} was picked

digits w_j are 0 or 1
only max 5 1's in any position
$\Longrightarrow$ no carries